The Schwarz type inequalities for harmonic mappings with boundary normalization

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Calculating procedure of the regression functions

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Analytic in the unit ball functions of bounded L-index in direction and everywhere dense in a sequence of balls set

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Let $\mathbf{b} = (b_1, \ldots, b_n) \in \mathbb{C}^n$ be a given direction, $\mathbb{B}^n = \{z \in \mathbb{C}^n : |z| < 1\}$, $L : \mathbb{B}^n \to \mathbb{R}_+$ be a continuous function such that for all $z \in \mathbb{B}^n$

$$L(z) > \frac{\beta|\mathbf{b}|}{1-|z|}, \ \beta = \mathrm{const} > 1, \mathbf{b} \in \mathbb{C}^n.$$

Analytic in $\mathbb{B}^n$ function $F(z)$ is called a function of *bounded L-index in a direction* $\mathbf{b} \in \mathbb{C}^n$, if there exists $m_0 \in \mathbb{Z}_+$ such that for every $m \in \mathbb{Z}_+$ and every $z \in \mathbb{B}^n$ the following inequality is valid

$$\frac{1}{m!L^m(z)} \left| \frac{\partial^m F(z)}{\partial \mathbf{b}^m} \right| \leq \max \left\{ \frac{1}{k!L^k(z)} \left| \frac{\partial^k F(z)}{\partial \mathbf{b}^k} \right| : 0 \leq k \leq m_0 \right\}, \tag{1}$$

where $\frac{\partial^0 F(z)}{\partial \mathbf{b}^0} = F(z)$, $\frac{\partial F(z)}{\partial \mathbf{b}} = \sum\limits_{j=1}^{n} \frac{\partial F(z)}{\partial z_j} b_j = \langle \mathbf{grad}\ F, \overline{\mathbf{b}} \rangle$, $\frac{\partial^k F(z)}{\partial \mathbf{b}^k} = \frac{\partial}{\partial \mathbf{b}} \left(\frac{\partial^{k-1} F(z)}{\partial \mathbf{b}^{k-1}} \right)$, $k \geq 2$.

Other definitions and denotations see in [1]. Our main result is following

Theorem 1. *Let (r_p) be a positive sequence such that $r_p \to 1$ as $p \to \infty$, $D_p = \{z \in \mathbb{C}^n : |z| = r_p\}$, A_p be an everywhere dense set in D_p (i.e. $\overline{A_p} = D_p$) and let $A = \bigcup\limits_{p=1}^{\infty} A_p$. Analytic in $\mathbb{B}^n$ function $F(z)$ is of bounded L-index in direction $\mathbf{b} \in \mathbb{C}^n$ if and only if there exists number $M > 0$ such that for all $z^0 \in A$ function $g_{z^0}(t) = F(z^0 + t\mathbf{b})$ is of bounded l_{z^0}-index $N(g_{z^0}, l_{z^0}) \le M < +\infty$, as a function of variable $t \in \mathbb{B}_{z^0}$, where $l_{z^0}(t) \equiv L(z^0 + t\mathbf{b})$. And $N_{\mathbf{b}}(F, L) = \max\{N(g_{z^0}, l_{z^0}) : z^0 \in A\}$.*

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A two-dimensiolal generalization of the Rutishauser QD-algorithm

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We propose an algorithm of calculation of the coefficients of a regular two-dimensional C-fraction with independent variables

$$\cfrac{a_0}{F_0(z_1) + \cfrac{a_{01}z_2}{F_1(z_1) + \cfrac{a_{02}z_2}{F_2(z_1) + \ddots}}}, \quad F_p(z_1) = 1 + \cfrac{a_{1p}z_1}{1 + \cfrac{a_{2p}z_1}{1 + \cfrac{a_{3p}z_1}{1 + \ddots}}},$$

where $a_0 \neq 0$, $a_{rs} \neq 0$, are complex-valued constants, $(z_1, z_2) \in \mathbb{C}^2$, the help of the corresponding given formal double power series

$$L(z_1, z_2) = \sum_{r,s=0}^{\infty} c_{rs} z_1^r z_2^s,$$

where $c_{rs} \in \mathbb{C}$, $(z_1, z_2) \in \mathbb{C}^2$, which is a two-dimensional generalization of the Rutishauser qd-algorithm ([1]). We establish the necessary and sufficient conditions for the existence of such algorithm and consider some examples of functions, which are represented by regular two-dimensional C-fractions with independent variables.

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The stability of solutions of the Cauchy problem for a differential-operator equation connected with oscillations stratified fluids

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We consider the equation

$$\varepsilon y^{IV}(t) + (A+E)y''(t) + Ay(t) = 0, \quad t \in [0,\infty), \tag{1}$$

where A is a selfadjoint lover semibounded operator in a Hilbert space H and $\varepsilon > 0$.

A vector-valued function $y(t)$ is called a solution of equation (1) if it is four times strongly differentiable in H and $y(t)$ satisfies (1).

A solution is called stable if

$$\sup_{t\in[0,\infty),k=0,1,2,3} ||y^{(k)}(t)|| < \infty.$$

If each solution is stable, then we say that the equation is stable.

Let $\sigma(A)$ be spectrum of the operator A and let $\gamma = \min\{0, \inf\limits_{||f||=1}(Af,f)\}$.

Theorem 1. *For arbitrary $f_i \in D(A), i = 0,1,2,$ and $f_3 \in D((A-\gamma E)^{\frac{1}{2}})$ there exists a unique solution of equation (1) which satisfies the initial conditions $y^{(k)}(0) = f_k, k = 0,1,2,3$.*

Theorem 2. *Equation (1) is stable if and only if*

$$(-\infty,0] \cap \sigma(A) \quad for \quad 0 < \varepsilon < 1,$$

$$\Big((-\infty,0] \cup [2\varepsilon - 1 - 2\sqrt{\varepsilon^2-\varepsilon}, 2\varepsilon - 1 + 2\sqrt{\varepsilon^2-\varepsilon}]\Big) \cap \sigma(A) = \varnothing \quad for \quad \varepsilon \geq 1.$$

The growth of Dirichlet series in the terms of generalized orders

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Let $A \in (-\infty, +\infty]$, Φ be a continuously differentiable convex function on $(-\infty, A)$, increasing together with Φ' to $+\infty$ on $(-\infty, A)$, φ be the inverse function to Φ', $\Psi(\sigma) = \sigma - \frac{\Phi(\sigma)}{\Phi'(\sigma)}$, $\lambda = (\lambda_n)$ be a sequence, increasing to $+\infty$, and $\mathcal{D}_A(\lambda)$ be the class of Dirichlet series of the form $F(s) = \sum a_n e^{s\lambda_n}$, absolutely convergent in the half-plane $\operatorname{Re} s < A$. We prove, that the equality

$$\varlimsup_{\sigma \uparrow A} \frac{\ln \sup\{|F(s)| : \operatorname{Re} s = \sigma\}}{\Phi(\sigma)} = \varlimsup_{n\to\infty} \frac{\lambda_n}{\Phi'\left(\Psi^{-1}\left(\frac{1}{\lambda_n} \ln \frac{1}{|a_n|}\right)\right)}$$

holds for every Dirichlet series $F \in \mathcal{D}_A(\lambda)$ if and only if

$$\ln n = o\left(\Phi\left(\varphi\left(\frac{\lambda_n}{B}\right)\right)\right), \quad n \to \infty,$$

for all $B > 0$.

Aggregation iterative algorithm for linear equations in a Banach space

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Problem of the operator equations decomposition is actual task. It is conditioned by the necessity of construction of methods of calculations parallelization. Multi-parameter iterative aggregation is an effective method of decomposition of the high dimension problems (see [1]).

Let E be a Banach space and $A : E \longmapsto E$ be a linear continuous operator. Consider the equation

$$x = Ax + b, \qquad b \in E. \tag{1}$$

For such equations often it is assumed that: 1) the normal cone $K \subset E$ of positive elements is given; 2) semiordering in E is introduced by such elements; 3) contraction operator A and element b are positive (see, for example, [2]). These and other requirements are caused by the specificity of the corresponding problems (see, for example, [3]). More detailed results for one-parametric method are given in [2, p. 155–158] and can be described by the formula

$$x^{(n+1)} = \frac{(\varphi, b)}{(\varphi, x^{(n)} - Ax^{(n)})} Ax^{(n)} + b \qquad (n = 0, 1, \ldots). \tag{2}$$

Here (φ, x) denotes the value of a linear functional $\varphi \in K^*$ on the elements $x \in E$, where K^* is a cone of positive elements in a dual Banach space E^*. The algorithm (2) is investigated in [2, p. 155–158] with the following assumptions: (i) A is a focusing operator [2, p. 77]; (ii) spectral radius $\rho(A)$ of the operator A is less than one; (iii) the functional φ is admissible. A functional φ is called an admissible if there exists a functional $g \in K^*$ such that $\varphi = A^* g$ and $(g, x) > (\varphi, x)$ for $x \in K$, $x \neq \Theta$, where A^* is conjugate to A operator and Θ is zero element in E.

In particular, if (1) is a system of linear algebraic equations with a matrix $A = \{a_{ij}\}$, then the focusing condition is valid when all a_{ij} are strictly positive numbers. For the linear integral operator of the following form

$$Ax = \int_a^b G(t, s) x(s) ds$$

the focusing condition is valid if the continuous function $G(t, s)$ satisfies the condition $G(t, s) \geqslant \varepsilon > 0$ for $t, s \in [a, b]$.

In [2, p. 158] it is noted that the theory of methods of iterative aggregation is not well developed and the conditions of their convergence are unknown. In particular, as it is indicated by numerous examples (see [2, p. 158]), one parametric method (2) often converges when the above conditions are not fulfilled.

In this work we investigate the multi-parameter algorithms of iterative aggregation using the methodology described in [4]. The established sufficient conditions of convergence do not contain the requirement of type $\rho(A) < 1$ for a spectral radius $\rho(A)$ of an operator A and the constant signs condition of an operator A and of the aggregating functionals.

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The spectra of algebras of block-symmetric analytic functions of bounded type

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Let $\mathcal{X}^2 = \oplus_{\ell_1}\mathbb{C}^2$ be an infinite ℓ_1-sum of copies of Banach space $\mathbb{C}^2$. A polynomial P on the space $\mathcal{X}^2$ is called block-symmetric (or vector-symmetric) if

$$P\left(\begin{pmatrix} u_1 \\ v_1 \end{pmatrix}_1, \ldots, \begin{pmatrix} u_m \\ v_m \end{pmatrix}_m, \ldots\right) = P\left(\begin{pmatrix} u_1 \\ v_1 \end{pmatrix}_{\sigma(1)}, \ldots, \begin{pmatrix} u_m \\ v_m \end{pmatrix}_{\sigma(m)}, \ldots\right),$$

for every permutation σ on the set $\mathbb{N}$, where $\begin{pmatrix} u_i \\ v_i \end{pmatrix} \in \mathbb{C}^2$. Let us denote by $\mathcal{P}_{vs}(\mathcal{X}^2)$ the algebra of block-symmetric polynomials on $\mathcal{X}^2$, $\mathcal{H}_{bvs}(\mathcal{X}^2)$ — the algebra of block-symmetric analytic functions of bounded type on $\mathcal{X}^2$ and $\mathcal{M}_{bvs}(\mathcal{X}^2)$ — the spectra of algebra $\mathcal{H}_{bvs}(\mathcal{X}^2)$.

In this report it will be described the spectra of algebras of block-symmetric analytic functions of bounded type on the ℓ_1-sum of $\mathbb{C}^2$ by exponential type functions of two complex variables.

Multiplicative analytic mappings on Banach spaces

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Let A be a Banach algebra with an unit e on the complex numbers field $\mathbb{C}$. It is well known that every linear multiplicative functional (a character) on an algebra A is automatically continuous. It is shown that a similar statement is true for multiplicative polynomials from A to $\mathbb{C}$ (see. [2]). Besides that every entire multiplicative function from A to $\mathbb{C}$ is an homogeneous polynomial.

In this report it will be described the approach to build multiplicative analytic functions in some areas of Banach algebra A. It will be presented the examples to show that such functions are not necessarily homogeneous polynomials. Besides that it will be built the frangible G-analytic multiplicative function in some open everywhere dense set of algebra A.

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Pointwise stabilization solutions for the equations of the diffusion type with inertia

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In this paper we consider the integral Poisson point stabilization for equations of diffusion of inertia with finite number of groups of variables which are degenerate diffusion. The problem of stabilization of solutions of the Cauchy problem for parabolic equations engaged S.D Eidelman, F. Porper, V.P. Repnikov. Necessary and sufficient conditions for stabilization point Poisson integral for the Kolmogorov set to S.D. Eidelman, V.P. Repnikov, G.P. Malitskaya [1], generalization of these results is the work of 2 in the case of 3-oh groups degeneration [2].

Let

$$x := (x_{11}, ..., x_{1n_1}; x_{21}, ..., x_{2n_2}; ...; x_{k1}, ..., x_{kn_k}; ...; x_{p1}, ..., x_{pn_p}; x_{p+1,1}, ..., x_{m1}),$$

$$n_1 \geq n_2 \geq ... \geq n_p > 1, n_k \in N,\ k = \overline{1,p},\ p \in N,$$

$$m \geq p,\ \sum_{k=1}^{p} n_k + m - p = n,\ \ x \in R^n,\ \xi \in R^n.$$

Consider the Cauchy problem

$$\partial_t u\,(t,x) - \sum_{k=1}^{p}\sum_{j=1}^{n_k} x_{kj}\partial_{x_{k\,j+1}} u\,(t,x) = \sum_{v=1}^{m} \partial^2_{x^2_{k1}} u\,(t,x)\,, \tag{1}$$

$$u\,(t,x)\,|_{t=\tau} = u_0\,(x)\,,\ \ 0 \leq \tau < t \leq T < +\infty,\ x \in R^n, \tag{2}$$

where $u_0\,(x)$– measurable by Lebesgue bounded function in R^n. FSCP (1), (2), $G(t-\tau,\,x,\,\xi)$ at $t > \tau, x \in R^n, \xi \in R^n$ found in [3]. $\rho(t,x;0,\xi) = r^2$ – family of surfaces f.r. problem (1), (2).

Through $F^{x,0}_{r,t}$ denote the body, limited ellipsoid

$$\rho(t,x;0,\xi) = r^2, \tag{3}$$

where ξ – variable point across v_n – volume of the body bounded by the surface. Let $M^x_t(r)$ - average $u_0(x)$ an incredible run $F^x_{r,t}$, bounded surfaces (3). Function $u_0(x)$ must limit average $M^x(r)$ an incredible run $F^x_{r,t}$, if there $\lim\limits_{t\to\infty} M^x_t(r) = M^x(r)$.

Theorem 1. *If $u_0(x)$ a limit on the average ellipsoids $F^{x,0}_{r,t}$, which almost all r is equal to $M^x(r)$, then Poisson integral equation* (1) *stabilized (directed at $t \to \infty$) to the number*

$$\iota = (2\pi)^{-n/2} v_n \int\limits_0^{+\infty} r^{n+1} e^{-r^2} M^x(r) dr.$$

Theorem 2. *If $u_0(x) \geq 0$, is to stabilize the Poisson integral $u(t,x) = \int\limits_{R^n} G(t,x;0,\xi)u_0(\xi)d\xi$ zero is necessary and sufficient to $u_0(x)$ had a mean threshold $M^x(r)$, which is almost everywhere equal to zero.*

Theorems of pointwise stabilization for Poisson integral equation of diffusion of inertia if there is a limit of the average primary measurable bounded functions can be ported to systems of equations with constant coefficients Kolmogorov. Stabilization Poisson integral equation (1) can be directly connected with resistance value of derivatives financial markets [4].

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The properties of free Banach spaces

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The concept of a free Banach space $B(X)$ over a metric space X such that every Lipschitz map from X to a normed space E can be extended to a continuous linear operator from $B(X)$ to E was introduced in [1]. This approach delivers us some kind of global linearization of Lipschitz maps where we can apply methods of Banach space theory. On the other hand it is interesting to study properties of $B(X)$ and construct free Banach spaces for given metric spaces X.

Let $f : X \to Y$ be a map between metric spaces. The modulus of continuity of f is the function

$$\omega_f(t) = \sup\{\rho_Y(f(x), f(y)) \ : \ x, y \in X \text{ and } \rho_X(x, y) \le t\}.$$

If for some constant c $\omega_f(t) \le ct$ for every $t > 0$, the map f is called a *Lipschitz map* and the smallest possible such constant is called its *Lipschitz constant.* The set of Lipschitz maps from X to Y is denoted by $\mathrm{Lip}(X, Y)$.

A *normed set* (X, ρ, α) is a metric space (X, ρ) with a non-negative real-valued function α such that $|\alpha(x) - \alpha(y)| \le \rho(x, y) \le \alpha(x) + \alpha(y)$ for all $x, y \in X$. We call α a *norm* on X. It is clear, that an arbitrary metric space X is a normed set with respect to the norm $\alpha(x) := \rho(\theta, x)$, where θ is some *marked* point in X. A metric space X with a fixed marked point is called a *pointed* space.

Let E be a normed space (i.e. normed linear space). Let us denote by $\mathrm{Lip}_0(X, E)$ the set of all Lipschitz maps from a normed set X with marked point θ and norm α to E such that $\|F(x)\| \le L_F\alpha(x)$ for every $F \in \mathrm{Lip}_0(X, E)$, where L_F is the Lipschitz constant of F. If $F \in \mathrm{Lip}(X, E)$, then $F \in \mathrm{Lip}_0(X, E)$, if and only if $F(\theta) = 0$. It is easy to see that $\mathrm{Lip}_0(X, E)$ is a normed space with norm $\|F\| = L_F$. The following theorem is known.

Theorem 1. *Let $X = (X, \rho, \alpha)$ be a normed set. There exists a unique, up to an isometric isomorphism, Banach space $B(X)$ over the field $\mathbb{K}$ and an isometric embedding $\nu : X \to B(X)$ such that*

1. *Vectors $\nu(x)$, $x \in X$ with $\alpha(x) > 0$ are linearly independent in $B(X)$ and span$\nu(X)$, the linear span of $\nu(x)$, $x \in X$ is dense in $B(X)$.*

2. *Every map F from $Lip_0(X,E)$ can be extended to a continuous linear operator $\widetilde{F}: B(X) \to E$ such that $\|\widetilde{F}\| = L_F$ for an arbitrary normed space E.*

The space $B(X)$ is called the *free Banach space* over X.

We say that a normed set X with a marked point θ is *normalized* if $\alpha(x) = 1$ for every $x \in X$, $x \neq \theta$.

Proposition 2. *Let X be a complete normed set such that $\alpha(x) \geq 1$ for every $x \in X$ with $x \neq \theta$. Then there is a closed subset X^η of the unit sphere of $B(X) \cup \theta$ such that $X^\eta \in \mathcal{N}$ and $B(X)$ is isometrically isomorphic to $B(X^\eta)$.*

Corollary 3. *Let X be a Banach space such that X is isomorphic to its hyperplane. Then there is a closed subset M_X of the unit sphere S_X of X such that*

1. *The space $B(X^+)$ is isomorphic to $B(M_X \cup 0)$.*
2. *The restriction of Lipschitz maps from $Lip(X)$ onto M_X is a Banach algebra that is isomorphic (as a Banach space) to $Lip(X)$.*
3. *The set M_X may be identified with set of weak star continuous scalar homomorphisms of this algebra.*

In [2] we consider other properties of $B(X)$ and construct it for some simple metric spaces.

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Homomorphisms and functional calculus on algebras of analytic functions on Banach spaces

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Let A be a commutative Banach algebra, X be a Banach space over the field of complex numbers $\mathbb{C}$.

Consider the complete projective tensor product $A \otimes_\pi X$. Every element of $A \otimes_\pi X$ can be represented by the form $\overline{a} = \sum_k a_k \otimes_\pi x_k$, where $a_k \in A,\ x_k \in X$. For every $\overline{a} \in A \otimes_\pi X$ and $f \in H_b(X)$ (algebra of entire analytic functions of bounded type on a Banach space X) let us define $\overline{f}(\overline{a})$ in the means of functional calculus for analytic functions on a Banach spaces ([5]). Then $\widetilde{\overline{f}}$ is the Aron-Berner extension of $\overline{f}$.

In [7] it is proved a theorem about a homomorphism of algebras $H_b(X)$ and $H_b((A \otimes_\pi X''), A)$ in the case when A is some finite dimensional algebra with identity. The following theorem extends this result for the case of an infinite dimensional algebra A.

Proposition 1. *Let A be the Arens regular Banach algebra. For every $f \in H_b(X)$ there exists a function $\widetilde{\overline{f}} \in H_b((A \otimes_\pi X)'', A'')$ such that $\widetilde{\overline{f}}(e \otimes x) = ef(x), x \in X$ and the mapping $F : f \mapsto \widetilde{\overline{f}}$ is a homomorphism between algebras $H_b(X)$ and $H_b((A \otimes_\pi X)'', A'')$.*

Next, we consider the case when A is a reflexive Banach algebra. Let us denote by $\mathcal{P}(^n X)$ the Banach space of all continuous n-homogeneous complex-valued polynomials on X. $\mathcal{P}_f(^n X)$ denotes the subspace of n-homogeneous polynomials of finite type, that is, the subspace generated by finite sum of finite products of linear continuous functionals. The closure of $\mathcal{P}_f(^n X)$ in the topology of uniform convergence on bounded sets is called the space of approximable polynomials and denoted by $\mathcal{P}_c(^n X)$.

Let us denote by $A_n(X)$ the closure of the algebra, generated by polynomials from $\mathcal{P}(^{\leqslant n} X)$ with respect to the uniform topology on bounded subsets of X. It is clear that $A_1(X) \cap \mathcal{P}(^n X) = \mathcal{P}_c(^n X)$.

Let us denote by $\mathcal{L}(H_b(X), A)$ the space of all continuous n-linear operators on $H_b(X)$ to A and let $M_A(H_b(X))$ be the set of all homomorphisms on $H_b(X)$ to A.

In [4] introduced a concept of radius function $R(\varphi)$ of a given linear functional $\varphi \in H_b(X)'$ as the infimum of all numbers $r > 0$ such that ϕ is bounded with respect to the norm of uniform convergence on the ball rB and proved that

$$R(\phi) = \limsup_{n\to\infty} \|\phi_n\|^{1/n},$$

where ϕ_n is the restriction of ϕ to $\mathcal{P}(^n X)$. In [7] extended this definition to a homomorphism $\Phi \in M_A(H_b(X))$, that is, $R(\Phi)$ is the infimum of all numbers $r > 0$ such that Φ is bounded with respect to the norm of uniform convergence on the ball rB and proved that

$$R(\Phi) = \limsup_{n\to\infty} \|\Phi_n\|^{1/n},$$

where Φ_n is the restriction of Φ to space n-homogeneous polynomials.

In the work [6] it was formulated and proved the Lemma 1 on extension of the linear functional $\varphi \in H_b(X)'$ to character $\psi \in M_b$. The following theorem is a generalization of the known lemma and is related to the study of extension of linear operator to the homomorphism.

Theorem 2. *Let $\Phi \in \mathcal{L}(H_b(A \otimes_\pi X), A)$ be a linear operator such that $\Phi(P) = 0$ for every $P \in \mathcal{P}(^m(A \otimes_\pi X), A) \cap A_{m-1}(A \otimes_\pi X)$, where m is a fixed positive integer and Φ_m be the nonzero restriction of Φ to $\mathcal{P}(^m(A \otimes_\pi X))$.*

Then there is a homomorphism $\Psi \in M_A(H_b(A \otimes_\pi X))$ such that its restrictions Ψ_k to $\mathcal{P}(^k(A \otimes_\pi X)$ satisfy the conditions: $\Psi_k = 0$ for all $k < m$ and $\Psi_m = \Phi_m$. Moreover, the radius functions of Ψ is calculated by the formula

$$\|\Phi_m\|^{1/m} \le R(\Psi) \le e\|\Phi_m\|^{1/m}.$$

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“Elementary” functional calculus for countable set of operators

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Denote by $\mathcal{S}_+ := \mathcal{S}(\mathbb{R}_+)$ Schwartz space of rapidly decreasing functions on $\mathbb{R}_+ := [0, +\infty)$. In the sequel, $\mathcal{S}_+^{\hat{\otimes}n}$ denotes symmetric projective tensor product of n copies of the space $\mathcal{S}_+$. Let by definition $\mathcal{S}_+^{\hat{\otimes}0} := \mathbb{C}$. Denote $\Gamma(\mathcal{S}_+) := \bigoplus_{fin \atop n\in\mathbb{Z}_+}(\mathcal{S}_+^{\hat{\otimes}n})$. In what follows elements of the space $\Gamma(\mathcal{S}_+)$ will be written as $\boldsymbol{p} = \bigoplus_{n=0}^m p_n$ for some $m \in \mathbb{N}$, where $p_n \in \mathcal{S}_+^{\hat{\otimes}n}$ for all $n \in \mathbb{Z}_+$, $p_0 \in \mathbb{C}$. To simplify, we write $\boldsymbol{p} = \big(p_n\big)$ instead of $\boldsymbol{p} = \bigoplus_{n=0}^m p_n$.

Let us define the operation $\boldsymbol{p}^{\otimes} \circledast \boldsymbol{q}^{\otimes} := \bigoplus_{n\in\mathbb{Z}_+}(\varphi * \psi)^{\otimes n}$ for any elements $\boldsymbol{p}^{\otimes} := (\varphi^{\otimes n})$ and $\boldsymbol{q}^{\otimes} := (\psi^{\otimes n})$, $\varphi, \psi \in \mathcal{S}_+$, from the total subset $\{\boldsymbol{p}^{\otimes} : \varphi \in \mathcal{S}_+\}$ of the space $\Gamma(\mathcal{S}_+)$ and extend this operation by linearity and continuity to whole space $\Gamma(\mathcal{S}_+)$. Note, that the space $\Gamma(\mathcal{S}_+)$ becomes an algebra under the operation $\circledast$.

Let $\widehat{\mathcal{S}}_+ := \mathcal{F}_+[\mathcal{S}_+]$ stand for the range of the space $\mathcal{S}_+$ under the Fourier transformation $\mathcal{F}_+$. Denote $\Gamma(\widehat{\mathcal{S}}_+) := \bigoplus_{n\in\mathbb{Z}_+} \widehat{\mathcal{S}}_+^{\hat{\otimes}n}$.

Now we can extend the Fourier transformation $\mathcal{F}_+$ onto the space $\Gamma(\mathcal{S}_+)$ as follows. First of all, for any $\varphi^{\otimes n} \in \mathcal{S}_+^{\hat{\otimes}n}$ with $\varphi \in \mathcal{S}_+$, we define the operation $\mathcal{F}_+^{\otimes n}$ by the relations $\mathcal{F}_+^{\otimes n} : \varphi^{\otimes n} \longmapsto \widehat{\varphi}^{\otimes n}$ and $\mathcal{F}_+^{\otimes 0} := \mathbf{1}_{\mathbb{C}}$, where $\mathbf{1}_{\mathbb{C}}$ is the operator of multiplication onto $1 \in \mathbb{C}$. Next, we extend $\mathcal{F}_+^{\otimes n}$ to whole space $\mathcal{S}_+^{\hat{\otimes}n}$ by linearity and continuity, so $\mathcal{F}_+^{\otimes n} \in \mathscr{L}\big(\mathcal{S}_+^{\hat{\otimes}n}, \widehat{\mathcal{S}}_+^{\hat{\otimes}n}\big)$. And finally, $\mathcal{F}_+^{\otimes}$ is defined to be the mapping

$$\mathcal{F}_+^{\otimes} : \Gamma(\mathcal{S}_+) \ni \boldsymbol{p} = \bigoplus_{n\in\mathbb{Z}_+} p_n \longmapsto \widehat{\boldsymbol{p}} := \bigoplus_{n\in\mathbb{Z}_+} \widehat{p}_n \in \Gamma(\widehat{\mathcal{S}}_+), \quad \text{where} \quad \widehat{p}_n := \mathcal{F}_+^{\otimes n} p_n.$$

Note, that $\widehat{\varphi}^{\otimes n}$ for any $n \in \mathbb{N}$ may be treated as a function of n variables $\mathbb{R}^n \ni (\xi_1, \ldots, \xi_n) \longmapsto \widehat{\varphi}(\xi_1) \cdot \ldots \cdot \widehat{\varphi}(\xi_n) \in \mathbb{C}$ and may be written in the following way

$$\widehat{\varphi}^{\otimes n}(\xi_1, \ldots, \xi_n) = \int_{\mathbb{R}_+^n} e^{-\mathrm{i}(t,\xi)_n} \varphi(t_1) \cdot \ldots \cdot \varphi(t_n)\, dt,$$

where $(t, \xi)_n := t_1\xi_1 + \cdots + t_n\xi_n$, $dt := dt_1 \ldots dt_n$. So, any element $\widehat{\boldsymbol{p}} \in \Gamma(\widehat{\mathcal{S}}_+)$ we may consider as a function of infinite many variables

$$\widehat{\boldsymbol{p}} : (\xi_1, \ldots, \xi_n, \ldots) \longmapsto \bigoplus_{n\in\mathbb{Z}_+} \widehat{p}_n(\xi_{\mathfrak{b}_n}, \ldots, \xi_{\mathfrak{e}_n}), \tag{1}$$

where $\mathfrak{b}_n := \frac{n(n-1)}{2} + 1$, $\mathfrak{e}_n := \frac{n(n+1)}{2}$.

Observe, that actually each function $\widehat{\boldsymbol{p}}$ depends on finite but not fixed number of variables.

Let F be a complex Banach space. Denote by A the countable system of generators of C_0-semigroups, i.e. $A := (A_1, A_2, \dots, A_n, \dots)$, where $-\mathrm{i}A_n$ generates a strong continuous contraction semigroup $0 \le t \longmapsto e^{-\mathrm{i}tA_n} \in \mathscr{L}(F)$ for all $n \in \mathbb{N}$. Let $\mathscr{G}$ be the set of such countable systems of generators, and $\mathscr{G}_n$ denote the set of each n collections of such generators. Let by definition $\mathscr{G}_0 = \{\emptyset\}$.

Let I_F stand for identity operator in F. Let $A_{(n)} := (A_{\mathfrak{b}_n}, \dots, A_{\mathfrak{e}_n})$, where $\mathfrak{e}_n = \frac{n(n+1)}{2}$, $\mathfrak{b}_n = 1+\mathfrak{e}_{n-1}$ for all $n \in \mathbb{N}$. Let by definition $A_{(0)} := \emptyset$. It follows, that $\mathbb{R}^n_+ \ni t \longmapsto e^{-\mathrm{i}t\cdot A_{(n)}} := e^{-\mathrm{i}t_1A_{\mathfrak{b}_n}} \circ \dots \circ e^{-\mathrm{i}t_nA_{\mathfrak{e}_n}} \in \mathscr{L}(F)$ is an n-parameter C_0-semigroup with the generator $A_{(n)}$, where we used the notation $t \cdot A_{(n)} := t_1A_{\mathfrak{b}_n} + \dots + t_nA_{\mathfrak{e}_n}$ for all $n \in \mathbb{N}$.

Consider the space of functions

$$\widetilde{\mathcal{S}}_n = \left\{\widetilde{p}_n \colon \mathscr{G}_n \longrightarrow \mathscr{L}(F) : p_n \in \mathcal{S}_+^{\hat{\otimes}n}\right\}, \qquad n \in \mathbb{Z}_+,$$

where

$$\widetilde{p}_n(A_{(n)}) := \int\limits_{\mathbb{R}^n_+} e^{-\mathrm{i}t\cdot A_{(n)}} p_n(t)\,dt, \qquad \text{for} \qquad n \in \mathbb{N}, \tag{2}$$

and $\widetilde{p}_0(A_{(0)}) := p_0 I_F$.

Define the mapping

$$\mathcal{L} := (\mathcal{L}_n) \colon \Gamma(\mathcal{S}_+) \ni \boldsymbol{p} = \big(p_n\big) \quad \longmapsto \quad \widetilde{\boldsymbol{p}} := \sum_{n\in\mathbb{Z}_+} \widetilde{p}_n \in \widetilde{\mathcal{S}},$$

where $\widetilde{\mathcal{S}} := \sum_{n\in\mathbb{Z}_+} \widetilde{\mathcal{S}}_n$.

The mapping $\mathcal{L} \colon \Gamma(\mathcal{S}_+) \longrightarrow \widetilde{\mathcal{S}}$ is a homomorphism of the algebra $\big\{\Gamma(\mathcal{S}_+), \circledast\big\}$ and algebra of operator valued functions defined on $\mathscr{G}$. On the other hand, the map $\mathcal{F}_+^{\otimes} : \Gamma(\mathcal{S}_+) \longrightarrow \Gamma(\widehat{\mathcal{S}}_+)$ is a homomorphism too [1]. So, we can treat the mapping

$$\mathcal{L} \circ (\mathcal{F}_+^{\otimes})^{-1} \colon \Gamma(\widehat{\mathcal{S}}_+) \longrightarrow \widetilde{\mathcal{S}}$$

as an "elementary" functional calculus. In other words, we understand the operator $\widetilde{\boldsymbol{p}}(\mathbf{A}) = \sum_{n\in\mathbb{Z}_+} \widetilde{p}_n(A_n) \in \mathscr{L}(E)$ as a "value" of a function $\widehat{\boldsymbol{p}}$ of infinite many variables (see (1)) at a countable system $\mathbf{A} := (\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_n, \dots) \in \mathscr{G}$ of generators of contraction C_0-semigroups.

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Operator image of convolution algebra of Roumieu ultradistributions with supports on $\mathbb{R}^n_+$

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In the paper [2] is proposed modification of Hille-Phillips formula [1] making it possible to extend the set of symbols of this calculus to the space of Schwartz distributions with supports on semiaxis. Further [4] we have extended this calculus on the set of generators of n-parametric operator semigroups. In this lecture we explain how to construct a vector analogue of operator calculus for generators of strongly continuous n-parametric semigroups of operators in the convolution algebra of Roumieu ultradistributions with supports in the positive n-dimensional cone.

For constructed operator calculus we can consider examples of calculating Dirac function for generator of n-parametric strongly continuous semigroup of operators and solve the problem of representation of multiplicative powers and derivatives for Dirac function from the generator of the semigroup of fractional integration. Similar examples for Schwartz distributions are described in detail in the paper [3].

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Symmetric regularity of projective tensor product of Banach spaces

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Let X, Y be a complex Banach spaces and $B : X \times X \to Y$ is a bilinear (symmetric) map. A map B is called *(symmetrically) regular*, if

$$\lim_{\alpha,\beta} B(x_\alpha, y_\beta) = \lim_{\beta,\alpha} B(x_\alpha, y_\beta),$$

where $(x_\alpha), (y_\beta) \subset X$ are nets, which converges in $*$-weak topology of the space X^{**}. A space X is called *(symmetrically) regular*, if every (symmetric) bilinear form on $X \times X$ is regular (see. [1]). A Banach algebra A is called *Aren's regular*, if a bilinear map $(x, y) \to xy$ is regular.

Theorem 1. *If $\lim_{\alpha,\beta} P(x_\alpha, y_\beta) \neq \lim_{\beta,\alpha} P(x_\alpha, y_\beta)$, where $(x_\alpha), (y_\beta) \subset X$ are polynomially convergent nets, $P \in ¶(^n X)$, then a Banach space*

$$\sum^n X := C \oplus X \oplus X \otimes_{s,\pi} X \oplus \ldots \oplus \overbrace{X \otimes_{s,\pi} \ldots \otimes_{s,\pi} X}^{n}$$

is not symmetrically regular.

Let consider a symmetric bilinear map B_P on the space $\sum^n X$ for some $2n$-homogeneous polynomial P on X such, that $B_P(1 + x + \cdots + x^{\otimes n}, 1 + y + \ldots + y^{\otimes n}) = P(x + y)$. This map is an example of 4-homogeneous polynomial P on ℓ_2 such, that $\lim_{\alpha,\beta} P(x_\alpha + y_\beta) \neq \lim_{\beta,\alpha} P(x_\alpha + y_\beta)$, so the space $\sum^n \ell_2$ is not symmetrically regular.

Let A be a Banach (commutative) algebra.

Theorem 2. *A symmetric projective tensor product $\bigotimes_{s,\pi}^n A$ is not symmetrically regular.*

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On the spaces of (p,q)-linear and of (p,q)-homogeneous mappings between complex linear spaces

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We show that the space of (p,q)-linear mappings between complex li near spaces is a proper subspace of the space of (p,q)-homogeneous with respect to the collection of arguments and $(p+q)$-linear in the real sense mappings. Also we construct a projector from the space of (p,q)-homogene ous with respect to the collection of arguments and $(p+q)$-linear in the real sense mappings onto the space of (p,q)-linear mappings.

Zero-sets of diagonal polynomials on Banach spaces

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Let X be a Banach space. A map $P_n : X \to \mathbb{C}$ is n degree continuous homogenious polynomials, $n > 0$ (n-homogeneous polynomial), if there is an n-linear mapping $B_n : \underbrace{X \times X \ldots X}_{n} \to \mathbb{C}$ such that $P_n(x) = B_n(x, x, \ldots, x)$ for all $x \in X$.

Let X is a separable Banach space with an unconditional basis $\{e_n\}$.

Let $P(x)$ is a diagonal polynomial of the form $P(x) = \sum\limits_{n=1}^{\infty} c_k x_k^n$, where c_k is a bounded sequence.

Let us denote by u_k the following vectors

$$u_k = c_1 e_{2k-1} + c_2^{\frac{1}{n}} e_{2k} + c_3^{\frac{1}{n}} e_{2k+1} + \ldots + c_n^{\frac{1}{n}} e_{2k+n-2},\ k = 1, \ldots, n,$$

where $c_1 = 1, c_1 + c_2 + \ldots + c_n = 0$. Next construct spaces U_k such thet:

$$U_k = \overline{\operatorname{span}}\{u_1, \ldots, u_n, \ldots\}. \tag{1}$$

In the report we will considered linear subspace form (1) in zero-sets of complex n-homogenous polynomials, including diagonal polynomials, described some properties of such subspaces.

Note that in [1] it was proved that in zero-set of an arbitrary polynomial on infinite-dimensional complex space contains an infinite-dimensional subspace. Subspace of complex polynomials studied by many authors (see a survey [2]).

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